Fuel-efficient Powered Descent Guidance on Planetary Bodies via Theory of Functional Connections

[1]Enrico Schiassi, [1]Roberto Furfaro, [2]Hunter Johnston, and [2]Daniele Mortari

[1]University of Arizona, USA

[2]Texas A&M University, USA

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Introduction: Overview

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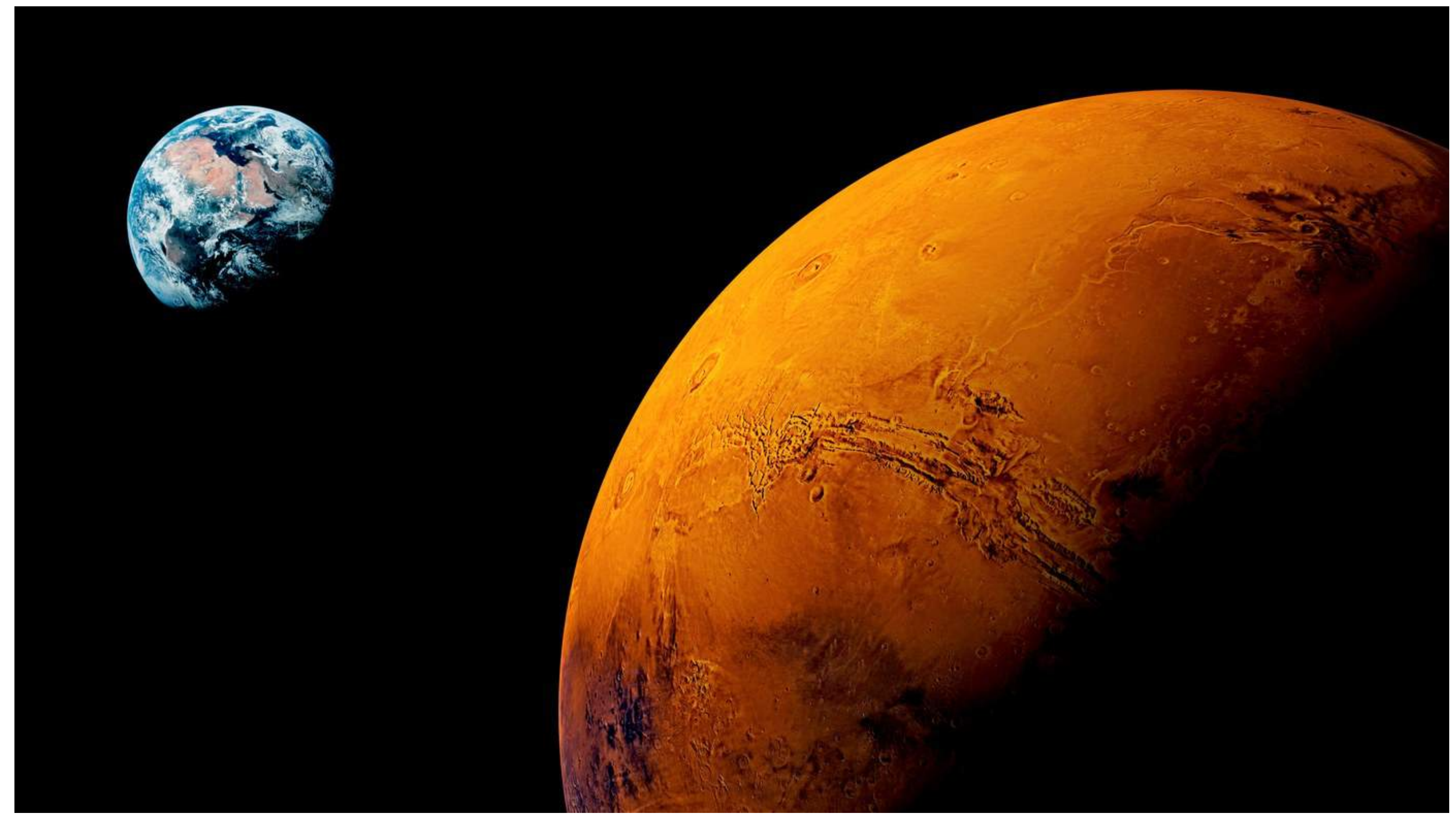
- Precision landing on planetary bodies is a key technology for future human and robotic exploration of the solar system
- To access planetary surfaces the landing system technology will need to progress to satisfy the demand for more stringent requirements
- Extremely important for precision landing is the ability to generate on-board and track in real-time fuel optimal trajectories



Introduction: Goals

- To develop a new algorithm, suitable for on-board application, based on the recently developed *Theory of Functional Connections* (TFC) [Mortari 2018] to compute fuel-efficient trajectories

- The focus of this talk is to show the capability of TFC in solving the equations of motion for the fuel-efficient powered descent guidance fast and with high accuracy

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Optimal Control for Space Guidance

- Optimal Control/Guidance is generally hard and computationally expensive
  - No direct analytical solutions except in very limited cases
- Open-Loop solutions can be generally found in two ways
  - **Direct Method**: Transform a continuous problem in a finite NLP problems and find the minimum
    - Convergence to a global minimum generally non-guaranteed
  - **Indirect Method**: Apply Pontryagin Minimum Principle (PMP) to derive the necessary conditions at the final time
    - Solve a TPBVP (generally not well-posed)
- Recently, there have been a great interest in solving optimal guidance problems in real-time
  - Close the loop by a sequence of open-loop solutions
  - Convexification approach: Solve a sequence of Convex Problems
    - Guarantee convergence to a global minimum in polynomial time

TFC approach to solving a TPBVP

- TFC derives expressions, called constrained expression, with an embedded set of *n* linear constraints

$$y(t) = g(t) + \sum_{k=1}^{n} \eta_k \, q_k(t) = g(t) + \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{q}(t)$$

- TFC has been successfully applied to solve linear [Mortari 2018], and nonlinear [Mortari and Johnston 2018] differential equations
  - Solutions computed via least-squares (iterative for the nonlinear case)
  - Machine error accuracy in milliseconds

TFC approach to solving a TPBVP

- Consider the following second-order boundary value problem:

$$F(t, y, \dot{y}, \ddot{y}) = 0 \quad \text{subject to:} \begin{cases} y(t_0) = y_0 \\ y(t_f) = y_f \end{cases} \tag{1}$$

- According to the literature [Mortari 2018] we have the following constrained expression:

$$y(t) = g(t) + \eta_1 + \eta_2\, t.$$

- By applying the constraints we get:

$$\eta_1 = \frac{1}{t_f - t_0}\Big[t_f\Big(y_0 - g_0)\Big) - t_0\Big(y_f - g_f)\Big)\Big]$$

$$\eta_2 = \frac{1}{t_f - t_0}\Big[\Big(y_f - g_f\Big) - \Big(y_0 - g_0)\Big)\Big],$$

TFC approach to solving a TPBVP

- Thus, the constrained expression and its derivatives become:

$$y(t) = g(t) + \frac{t_f - t}{t_f - t_0}(y_0 - g_0) + \frac{t - t_0}{t_f - t_0}(y_f - g_f) \qquad \dot{y}(t) = \dot{g}(t) - \frac{1}{t_f - t_0}(y_0 - g_0) + \frac{1}{t_f - t_0}(y_f - g_f) \qquad \ddot{y}(t) = \ddot{g}(t)$$

- By plugging into equation (1) we get:

$$\tilde{F}\left(t, g, \dot{g}, \ddot{g}\right) = 0. \quad (2)$$

- By discretizing the differential equation domain and defining the free function *g(t)* as some known basis function (Chebyshev polynomials for this work) with unknown coefficients, $g(t) = \xi^T h\big(x(t)\big)$, equation (2) reduces to:

$$\tilde{F}(\boldsymbol{\xi}) = 0 \qquad (3)$$

- (3) is a loss function that is solved for ξ via different optimization schemes
  - Iterative least-square (ILS) for nonlinear differential equations

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Optimal Powered Descent Pinpoint Landing

- The system dynamics during the power descent on large bodies is given by:

$$\dot{\boldsymbol{r}} = \boldsymbol{v}$$
$$\dot{\boldsymbol{v}} = \boldsymbol{a}_g + \frac{\boldsymbol{T}}{m}$$
$$\dot{m} = -\alpha T$$

Differential Equations

subject to

BCs

$$\boldsymbol{r}(0) = \boldsymbol{r}_0 \qquad \boldsymbol{r}(t_f) = \boldsymbol{r}_f$$
$$\boldsymbol{v}(0) = \boldsymbol{v}_0 \qquad \boldsymbol{v}(t_f) = \boldsymbol{v}_f$$
$$m(0) = m_0$$

Thrust

$$\boldsymbol{T} = T\,\hat{\boldsymbol{t}}$$
$$0 \leq T_{min} \leq T \leq T_{max}$$
$$||\hat{\boldsymbol{t}}|| = 1$$

Constraints

- The goal is to minimize the mass of propellant:

$$\underset{T, t_f}{\text{minimize}} \quad \alpha \int_0^{t_f} T \, \mathrm{d}\tau$$

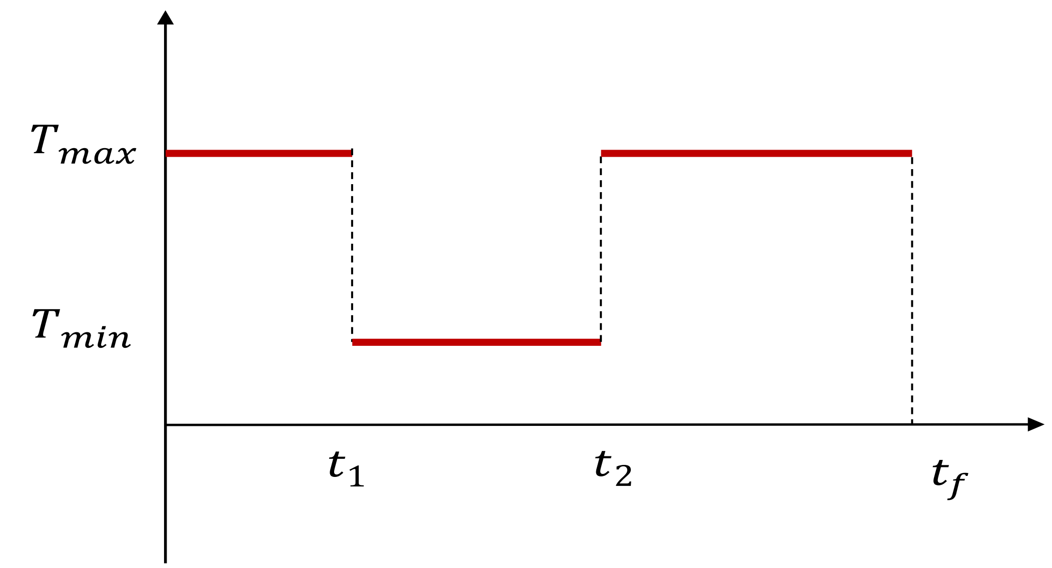
where $\boldsymbol{\alpha}$ is the reciprocal of the effective exhaust velocity of the rocket engine

Optimal Powered Descent Pinpoint Landing

- The necessary conditions for the optimal control problem calls for the Hamiltonian

$$H = \alpha T + \boldsymbol{\lambda}_r^{\mathrm{T}} \boldsymbol{v} + \boldsymbol{\lambda}_v^{\mathrm{T}} \left(\boldsymbol{a}_g + \frac{T}{m} \hat{\boldsymbol{t}} \right) - \lambda_m \alpha T$$

- From the optimal control theory is proved that:
  - The thrust direction is: $\hat{\boldsymbol{t}} = -\dfrac{\boldsymbol{\lambda}_v}{||\boldsymbol{\lambda}_v||}$
  - The thrust profile is bang-bang



Thrust profile [Ping Lu 2017]

- From the Hamiltonian we derive the co-state equations

$$\dot{\boldsymbol{\lambda}}_r = -\frac{\partial H}{\partial \boldsymbol{r}} = \boldsymbol{0}$$

$$\dot{\boldsymbol{\lambda}}_v = -\frac{\partial H}{\partial \boldsymbol{v}} = -\boldsymbol{\lambda}_r$$

$$\dot{\lambda}_m = -\frac{\partial H}{\partial m} = -\frac{T}{m^2} ||\boldsymbol{\lambda}_v||$$

Optimal Powered Descent Pinpoint Landing

- The whole TPBVP becomes:

$$\begin{aligned}
\dot{\boldsymbol{r}} &= \boldsymbol{v} \\
\dot{\boldsymbol{v}} &= \boldsymbol{a}_g - \frac{T(t;t_1,t_2)}{m(t)}\frac{\boldsymbol{\lambda}_v}{||\boldsymbol{\lambda}_v||} \\
\dot{m} &= -\alpha T(t;t_1,t_2) \\
\dot{\boldsymbol{\lambda}}_r &= \boldsymbol{0} \\
\dot{\boldsymbol{\lambda}}_v &= -\boldsymbol{\lambda}_r \\
\dot{\lambda}_m &= -\frac{T(t;t_1,t_2)}{m^2}||\boldsymbol{\lambda}_v||
\end{aligned}$$

subject to

$$\begin{aligned}
\boldsymbol{r}(0) &= \boldsymbol{r}_0 \\
\boldsymbol{v}(0) &= \boldsymbol{v}_0 \\
m(0) &= m_0 \\
\boldsymbol{r}(t_f) &= \boldsymbol{r}_f \\
\boldsymbol{v}(t_f) &= \boldsymbol{v}_f \\
\lambda_m(t_f) &= 0
\end{aligned}$$

- Two equations are redundant once the thrust profile is known:

$$\begin{aligned}
\dot{\boldsymbol{r}} &= \boldsymbol{v} \\
\dot{\boldsymbol{v}} &= \boldsymbol{a}_g - \beta(t)\frac{\boldsymbol{\lambda}_v}{||\boldsymbol{\lambda}_v||} \\
\dot{\boldsymbol{\lambda}}_r &= \boldsymbol{0} \\
\dot{\boldsymbol{\lambda}}_v &= -\boldsymbol{\lambda}_r
\end{aligned}$$

subject to

$$\begin{aligned}
\boldsymbol{r}(0) &= \boldsymbol{r}_0 \\
\boldsymbol{v}(0) &= \boldsymbol{v}_0 \\
\boldsymbol{r}(t_f) &= \boldsymbol{r}_f \\
\boldsymbol{v}(t_f) &= \boldsymbol{v}_f
\end{aligned}$$

TPBVP to be solved via TFC

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Solution of the Equations of Motions via TFC: Formulation

- TFC constrained expressions are analytical expressions, therefore:
  - The derivative of the constrained expression for $\boldsymbol{r}(t)$ is exactly $\boldsymbol{v}(t)$
  - The derivative of the constrained expression for $\boldsymbol{\lambda}_v(t)$ is exactly $-\boldsymbol{\lambda}_r(t)$

- The TPBVP further simplifies to a single differential equation:

$$\mathcal{L}_i = a_i - a_{g_i} + \beta(t) \frac{\lambda_{v_i}}{\left(\sum_{j=1}^{3} \lambda_{v_j}^2\right)^{\frac{1}{2}}}, \quad \text{for} \quad i = 1, 2, 3 \qquad (4)$$

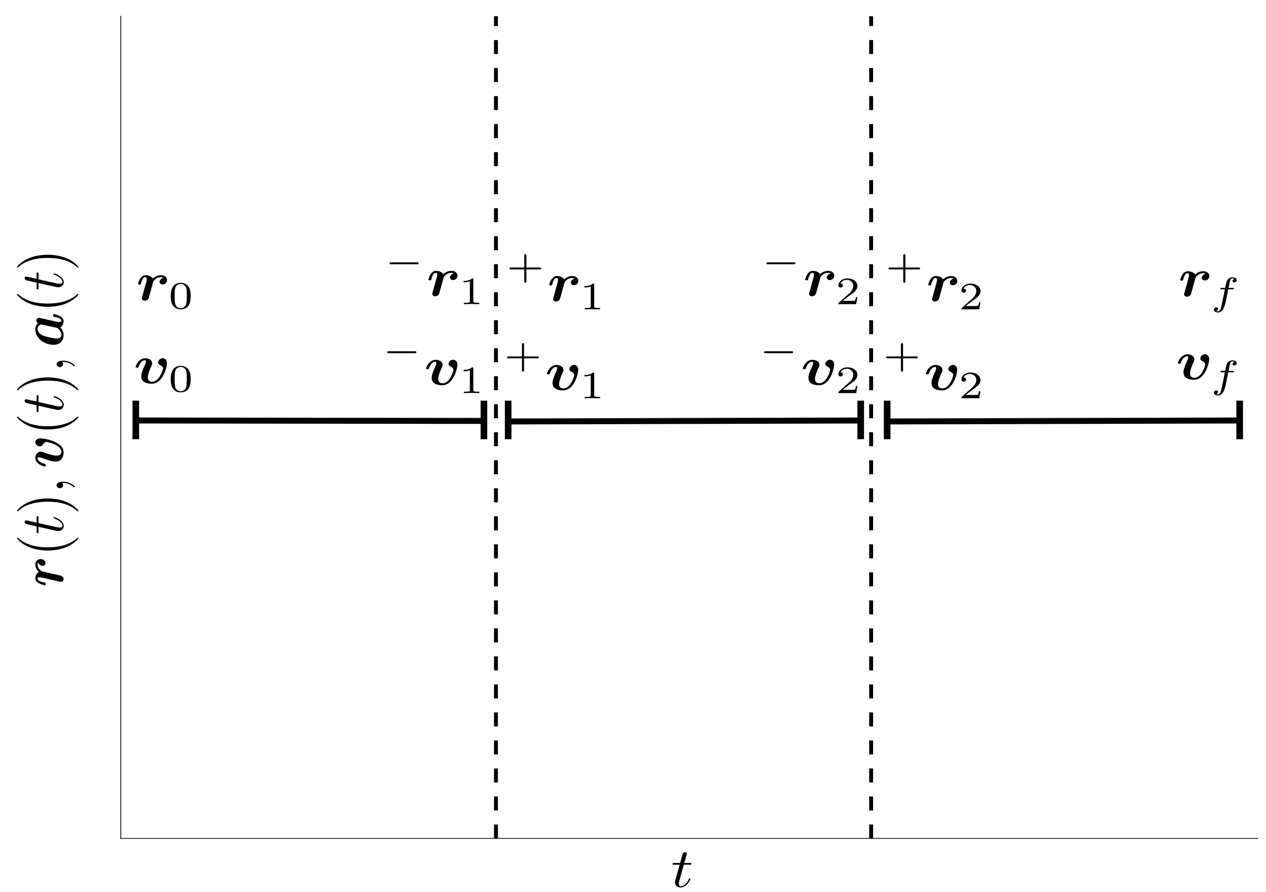
where : $a_i = \ddot{g}_i + \ddot{\Omega}_1 (r_{0_i} - g_{0_i}) + \ddot{\Omega}_2 (r_{f_i} - g_{f_i}) + \ddot{\Omega}_3 (v_{0_i} - \dot{g}_{0_i}) + \ddot{\Omega}_4 (v_{f_i} - \dot{g}_{f_i})$, $g_i = \boldsymbol{h}^{\mathrm{T}} \boldsymbol{\xi}_i$

$$\lambda_{v_i} = a_{0_i} + a_{1_i} t = \boldsymbol{h}_\lambda^{\mathrm{T}} \boldsymbol{\xi}_{\lambda_i}$$

- The Ω parameters are solely function of t and are switching functions (derived through TFC) to force the expression to always satisfy the constraints

Solution of the Equations of Motions via TFC: Formulation

- Due to the thrust bang-bang profile the TPBVP needs to be solved via the **TFC piecewise approach** [Johnston and Mortari *in progress*]:
  - The domain $[t_0, t_f]$ is split into three segments
    - Three distinct differential equations like (4) , governing the dynamic in each domain, must be solved simultaneously



- In each segment *s (s = 1, 2, 3)* the dynamic is regulated by the following equation:

$$^{(s)}\mathcal{L}_i = {}^{(s)}a_i - a_{g_i} + \beta(t)\frac{\lambda_{v_i}}{\left(\sum_{j=1}^{3}\lambda_{v_j}^2\right)^{\frac{1}{2}}}$$

- The embedded relative constraints allow:
  - the continuity of position and velocity, between each segment
  - the jumps of the acceleration, between each segment
- The embedded relative are unknowns that will be computed via the ILS

Solution of the Equations of Motions via TFC: Formulation

- A loss function for each segment *s (s = 1, 2, 3)* and each component *i (i =1, 2, 3)* need to be defined
- For each loss function we need to take the partial derivative for each unknown
- By discretizing the each subdomain in *N* points we get:

$$\mathbb{L} = \begin{Bmatrix} {}^{(1)}\mathcal{L}_1 \\ {}^{(1)}\mathcal{L}_2 \\ {}^{(1)}\mathcal{L}_3 \\ {}^{(2)}\mathcal{L}_1 \\ {}^{(2)}\mathcal{L}_2 \\ {}^{(2)}\mathcal{L}_3 \\ {}^{(3)}\mathcal{L}_1 \\ {}^{(3)}\mathcal{L}_2 \\ {}^{(3)}\mathcal{L}_3 \end{Bmatrix}_{(9N\times 1)} \qquad \Xi = \begin{Bmatrix} {}^{(1)}\boldsymbol{\xi}_1 \\ {}^{(1)}\boldsymbol{\xi}_2 \\ {}^{(1)}\boldsymbol{\xi}_3 \\ \boldsymbol{r}_1 \\ \boldsymbol{v}_1 \\ {}^{(2)}\boldsymbol{\xi}_1 \\ {}^{(2)}\boldsymbol{\xi}_2 \\ {}^{(2)}\boldsymbol{\xi}_3 \\ \boldsymbol{r}_2 \\ \boldsymbol{v}_2 \\ {}^{(3)}\boldsymbol{\xi}_1 \\ {}^{(3)}\boldsymbol{\xi}_2 \\ {}^{(3)}\boldsymbol{\xi}_3 \\ \boldsymbol{\xi}_{\lambda_1} \\ \boldsymbol{\xi}_{\lambda_2} \\ \boldsymbol{\xi}_{\lambda_3} \end{Bmatrix}_{(9m+18)}$$

$$\mathbb{J} = \begin{bmatrix} {}^{(1)}J_{\boldsymbol{\xi}} & {}^{(1)}J_{r_1,v_1} & \mathbf{0}_{(3N\times 3m)} & \mathbf{0}_{(3N\times 6)} & \mathbf{0}_{(3N\times 3m)} & {}^{(1)}J_{\boldsymbol{\xi}_\lambda} \\ \mathbf{0}_{(3N\times 3m)} & {}^{(2)}J_{r_1,v_1} & {}^{(2)}J_{\boldsymbol{\xi}} & {}^{(2)}J_{r_2,v_2} & \mathbf{0}_{(3N\times 3m)} & {}^{(2)}J_{\boldsymbol{\xi}_\lambda} \\ \mathbf{0}_{(3N\times 3m)} & \mathbf{0}_{(3N\times 6)} & \mathbf{0}_{(3N\times 3m)} & {}^{(3)}J_{r_2,v_2} & {}^{(3)}J_{\boldsymbol{\xi}} & {}^{(3)}J_{\boldsymbol{\xi}_\lambda} \end{bmatrix}_{(9N\times\{9m+18\})}$$

$$\boxed{\Xi_{k+1} = \Xi_k - \left(\mathbb{J}_k^{\mathrm{T}}\mathbb{J}_k\right)^{-1}\mathbb{J}_k^{\mathrm{T}}\mathbb{L}_k}$$

Iterative least-square

Solution of the Equations of Motions via TFC: Results

- Accuracy and convergence of TFC in solving the equations of motion are tested for a specific vehicle for Mars landing

$$v_{ex} = 2207.250 \text{ [m/s]}, T_{min} = 4500 \text{ [N]}, \text{ and } T_{max} = 12000 \text{ [N]}$$

Vehicle specifics

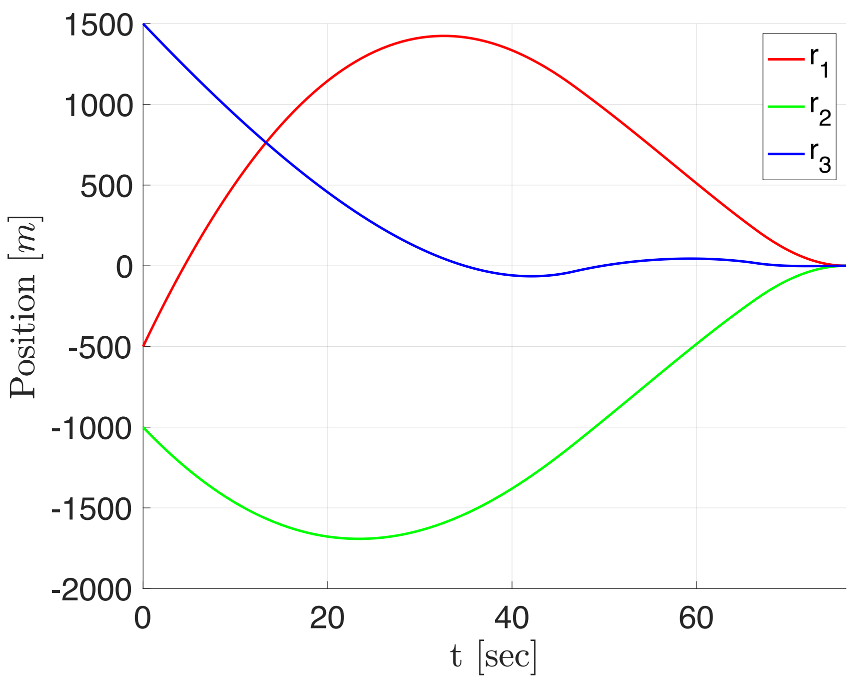
Variable	Initial	Final
$\boldsymbol{r}$ [m]	$\{-500, \quad -1000, \quad 1500\}^{\mathrm{T}}$	$\{0, \quad 0, \quad 0\}^{\mathrm{T}}$
$\boldsymbol{v}$ [m/s]	$\{120, \quad -60, \quad -60\}^{\mathrm{T}}$	$\{0, \quad 0, \quad 0\}^{\mathrm{T}}$
m [kg]	1905.00	-

State boundary conditions

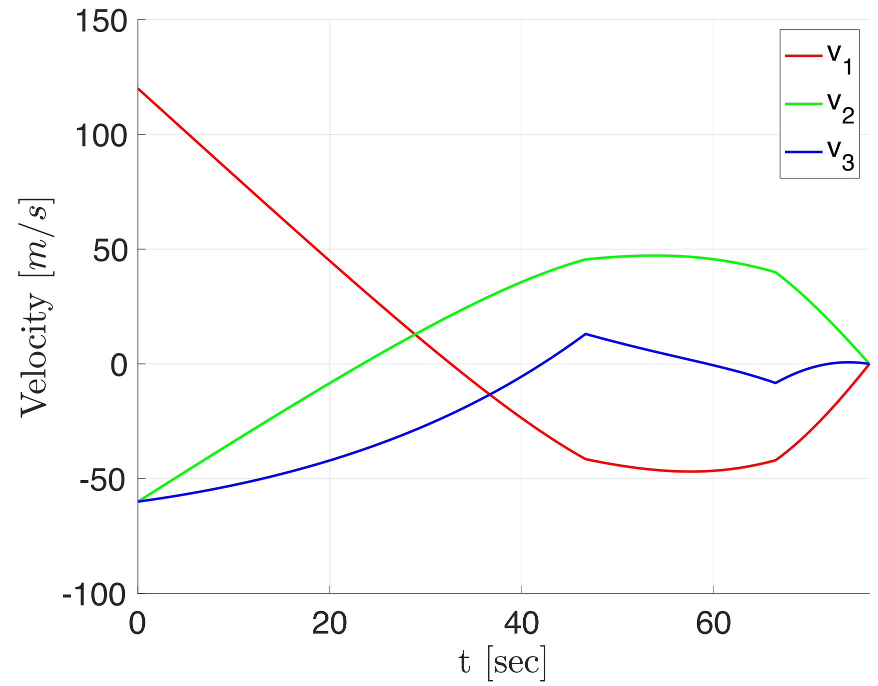
Variable	Value
Points per segment (N)	100
Number of basis functions per segment (m)	60
Convergence criteria of loss function: $L_2[\mathbb{L}] < \epsilon$	10^{-12}

TFC parameters for the numerical test

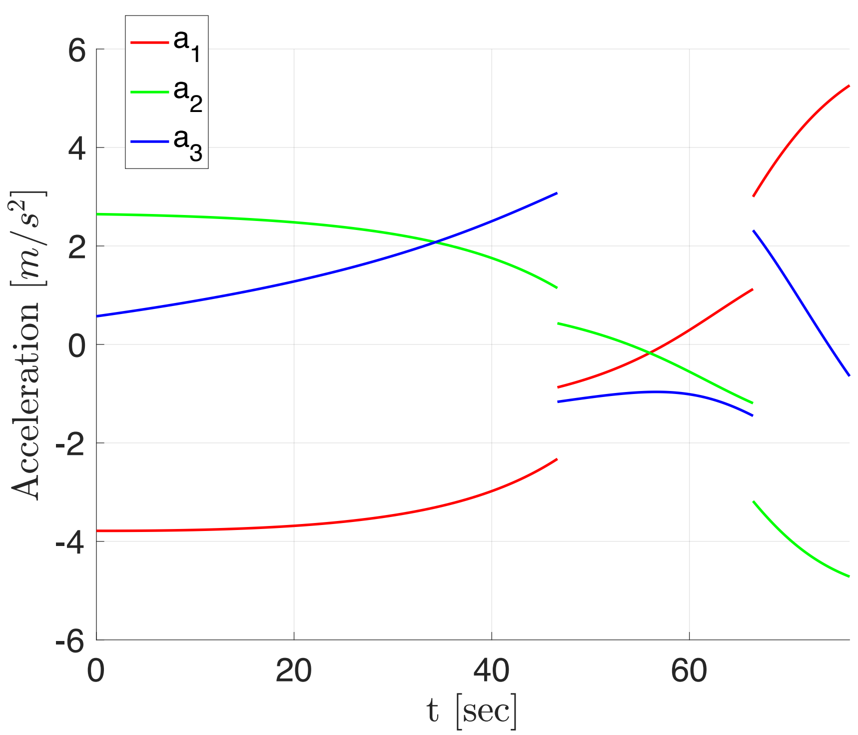
Solution of the Equations of Motions via TFC: Results



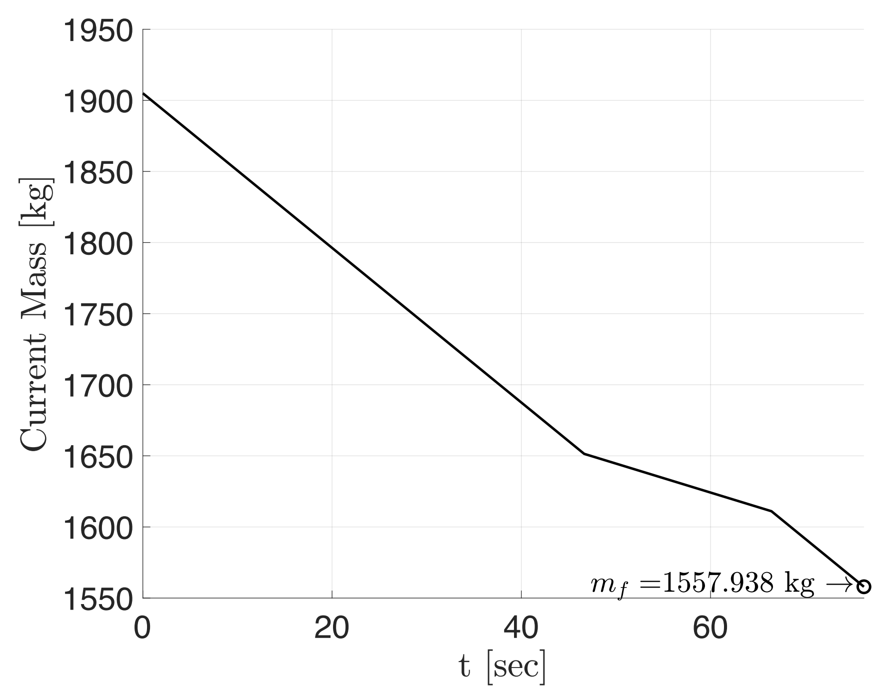
(a) Time history of the position.



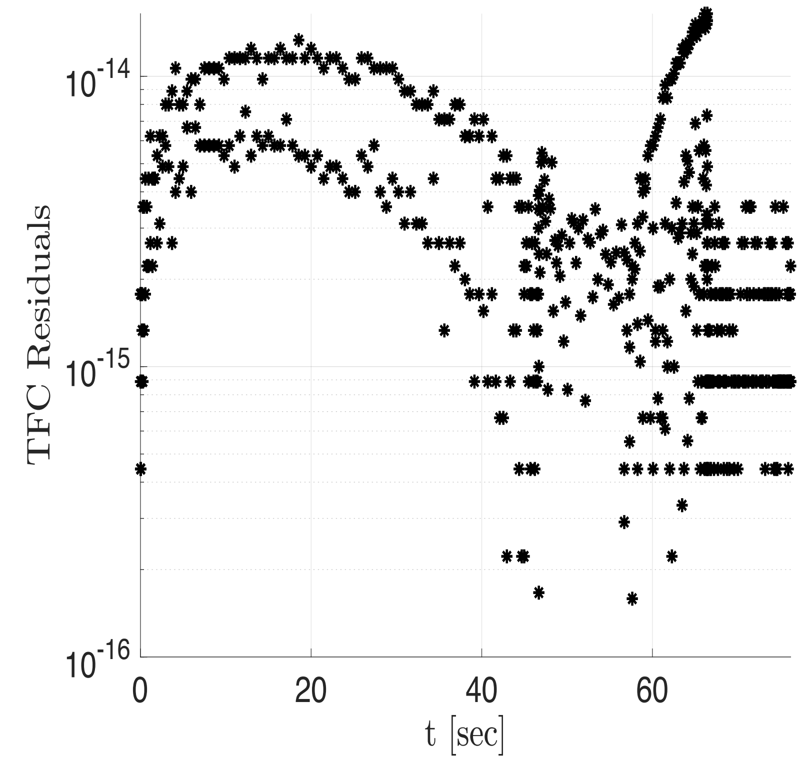
(b) Time history of the velocity.



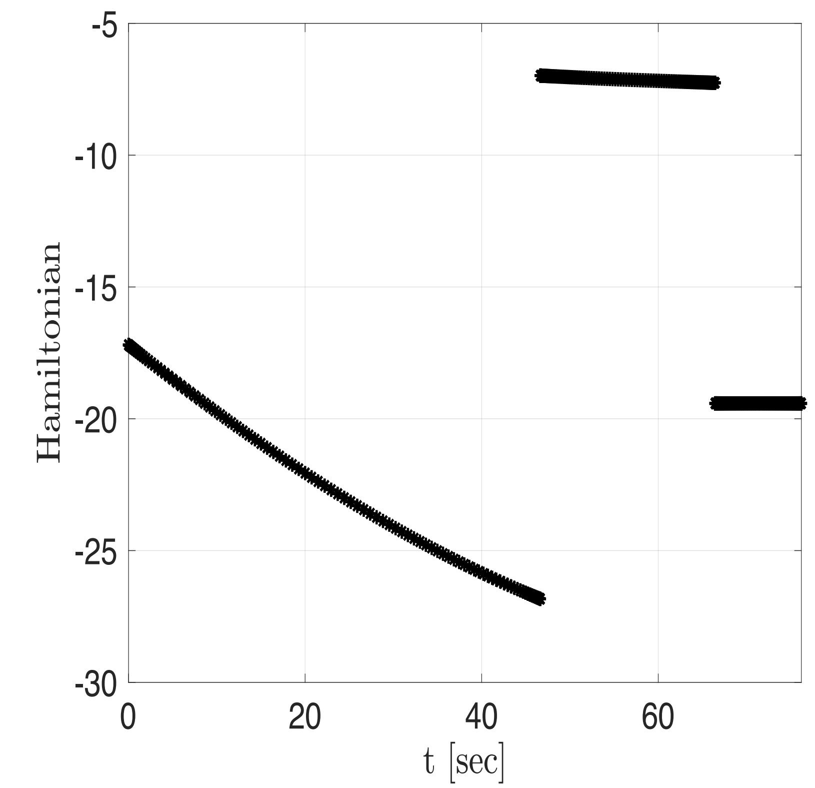
(c) Time history of the acceleration.



(d) Propellant mass over trajectory.



(a) Residual of governing differential equation.



(b) Time evolution of the Hamiltonian.

Iterations	10
$L_2[\mathbb{L}]$	1.66×10^{-13}

Convergence in only 10 iterations !!!

Would improve with a better first guess for the coefficients

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Conclusions and Outlooks

- TFC was successfully applied to solve the equations of motion for the fuel-efficient powered descent guidance
  - Machine error accuracy
  - Convergence achieved with only ten iterations
- Accurate trajectories, but still suboptimal as the condition for the free-time problem is not yet met
  - The switching times and the final time are suboptimal
  - The propellant used is not yet optimal
- Work in progress to develop the outer loop to find the optimal times that minimize the use of propellant such that
  - $L_2[\mathbb{L}] < \epsilon$
  - $H(t_f) = 0$

Questions ???

Thanks for the attention